# Wednesday 29 June 2016 - Morning <br> A2 GCE MATHEMATICS 

4734/01 Probability \& Statistics 3

## QUESTION PAPER

Candidates answer on the Printed Answer Book.
OCR supplied materials:

- Printed Answer Book 4734/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator


## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.


## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of $\mathbf{1 2}$ pages. The Question Paper consists of $\mathbf{4}$ pages. Any blank pages are indicated.


## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer all the questions.
1 On a motorway, lorries pass an observation point independently and at random times. The mean number of lorries travelling north is 6 per minute and the mean number travelling south is 8 per minute. Find the probability that at least 16 lorries pass the observation point in a given 1-minute period.

2 A random sample of 200 American voters were asked about which political party they supported and their attitude to a proposed new form of taxation. The voters' responses are summarised in the table.

| Attitude |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
|  |    <br>  In favour Neutral <br>    <br>  Democrat 58 <br> 16 16  <br>  Independent 25 <br> 4 11  <br>  Republican 17 <br> 20 33  |  |  |  |

Carry out a $\chi^{2}$ test, at the $1 \%$ level of significance, to investigate whether there is an association between party supported and attitude to the proposed form of taxation.

3 (i) A company packages butter. Of 50 randomly selected packs, 8 were found to have damaged wrappers. Find an approximate $95 \%$ confidence interval for the proportion of packs with damaged wrappers. [4]
(ii) The mass of a pack has a normal distribution with standard deviation 8.5 g . In a random sample of 10 packs the masses, in g , are as follows.

| 220 | 225 | 218 | 223 | 224 | 220 | 229 | 228 | 226 | 228 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Find a $99 \%$ confidence interval for the mean mass of a pack.

4 A group of students were tested in geography before and after a fieldwork course. The marks of 10 randomly selected students are shown in the table.

| Student | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ | $J$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mark before fieldwork | 19 | 84 | 84 | 99 | 59 | 19 | 29 | 49 | 54 | 69 |
| Mark after fieldwork | 23 | 98 | 83 | 88 | 68 | 33 | 28 | 53 | 58 | 88 |

(i) Use a suitable $t$-test, at the $5 \%$ level of significance, to test whether the students' performance has improved.
(ii) State the necessary assumption in applying the test.

5 The independent random variables $X$ and $Y$ have distributions $\mathrm{N}\left(30, \sigma^{2}\right)$ and $\mathrm{N}\left(20, \sigma^{2}\right)$ respectively. The random variable $a X+b Y$, where $a$ and $b$ are constants, has the distribution $\mathrm{N}\left(410,130 \sigma^{2}\right)$.
(i) Given that $a$ and $b$ are integers, find the value of $a$ and the value of $b$.
(ii) Given that $\mathrm{P}(X>Y)=0.966$, find $\sigma^{2}$.

6 The masses at birth, in kg , of random samples of babies were recorded for each of the years 1970 and 2010. The table shows the sample mean and an unbiased estimate of the population variance for each year.

| Year | No. of babies | Sample <br> mean | Unbiased estimate of <br> population variance |
| :---: | :---: | :---: | :---: |
| 1970 | 285 | 3.303 | 0.2043 |
| 2010 | 260 | 3.352 | 0.2323 |

(i) A researcher tests the null hypothesis that babies born in 2010 are 0.04 kg heavier, on average, than babies born in 1970, against the alternative hypothesis that they are more than 0.04 kg heavier on average. Show that, at the $5 \%$ level of significance, the null hypothesis is not rejected.
(ii) Another researcher chooses samples of equal size, $n$, for the two years. Using the same hypothesis as in part (i), she finds that the null hypothesis is rejected at the $5 \%$ level of significance. Assuming that the sample means and unbiased estimates of population variance for the two years are as given in the table, find the smallest possible value of $n$.

7 A continuous random variable $X$ has probability density function

$$
\mathrm{f}(x)=\left\{\begin{array}{cl}
a x^{3} & 0 \leqslant x \leqslant 1 \\
a x^{2} & 1<x \leqslant 2 \\
0 & \text { otherwise }
\end{array}\right.
$$

where $a$ is a constant.
(i) Show that $a=\frac{12}{31}$.
(ii) Find $\mathrm{E}(X)$.

It is thought that the time taken by a student to complete a task can be well modelled by $X$. The times taken by 992 randomly chosen students are summarised in the table, together with some of the expected frequencies.

| Time | $0 \leqslant x<0.5$ | $0.5 \leqslant x<1$ | $1 \leqslant x<1.5$ | $1.5 \leqslant x \leqslant 2$ |
| :--- | :---: | :---: | :---: | :---: |
| Observed frequency | 8 | 92 | 279 | 613 |
| Expected frequency | 6 | 90 |  |  |

(iii) Find the other expected frequencies and test, at the $5 \%$ level of significance, whether the data can be well modelled by $X$.

8 The radius, $R$, of a sphere is a random variable with a continuous uniform distribution between 0 and 10 .
(i) Find the cumulative distribution function and probability density function of $A$, the surface area of the sphere.
(ii) Find $\mathrm{P}(\mathrm{A} \leqslant 200 \pi)$.

## END OF QUESTION PAPER

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